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The μ -Problem in Theories with Gauge-Mediated Supersymmetry Breaking

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Abstract

We point out that the μ -problem in theories in which supersymmetry breaking is communicated to the observable sector by gauge interactions is more severe than the one encountered in the conventional gravity-mediated scenarios. The difficulty is that once μ is generated by a one-loop diagram, then usually B_μ is also generated at the same loop order. This leads to the problematic relation $B_\mu \sim \mu \Lambda$, where $\Lambda \sim 10\text{--}100$ TeV is the effective supersymmetry-breaking scale. We present a class of theories for which this problem is naturally solved. Here, without any fine tuning among parameters, μ is generated at one loop, while B_μ arises only at the two-loop level. This mechanism can naturally lead to an interpretation of the Higgs doublets as pseudo-Goldstone bosons of an approximate global symmetry.

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1 Introduction

During the early 1980's there was some considerable effort in developing theories with supersymmetry breaking originating in some hidden sector and then communicating with the observable sector via gauge interactions at the quantum level [1]. The goal was to construct realistic models which could circumvent the obstacle imposed by the tree-level mass sum rule in global supersymmetry [2]. Since then, these models have largely been abandoned in favour of the more promising theories where gravity mediates the supersymmetry breaking [3]. However, the puzzle of explaining the suppression of Flavour-Changing Neutral-Current (FCNC) processes in supergravity theories have recently revived the interest [4, 5] in models with Gauge-Mediated Supersymmetry Breaking (GMSB). Indeed, in this class of theories, the FCNC problem is naturally solved as gauge interactions provide flavour-symmetric supersymmetry-breaking terms in the observable sector.

In this paper we want to point out that in general GMSB theories suffer from a μ -problem. Usually one refers to the μ -problem as the difficulty in generating the correct mass scale for the Higgs bilinear term in the superpotential

$$W = \mu \bar{H} H , \quad (1)$$

which, for phenomenological reasons, has to be of the order of the weak scale.

In supergravity, if the term in eq. (1) is forbidden in the limit of exact supersymmetry, it is then generated at the correct scale as an effect of supersymmetry breaking, as long as the Kähler metric has not a minimal form [6]. The μ -problem in GMSB theories is, in a certain way, more severe. Suppose that the term in eq. (1) is forbidden in the limit of exact supersymmetry. As we will show in sect. 3, it is not difficult to envisage new interactions which generate μ at one loop, $\mu \sim \frac{\lambda^2}{(4\pi)^2} \Lambda$; here λ is some new coupling constant and Λ is the effective scale of supersymmetry breaking. It is fairly generic that the same interactions generate also the other soft supersymmetry-breaking terms in the Higgs potential at the same one-loop level, $m_H^2 \sim m_{\bar{H}}^2 \sim B_\mu \sim \frac{\lambda^2}{(4\pi)^2} \Lambda^2$, where

$$V_{\text{soft}} = m_H^2 |H|^2 + m_{\bar{H}}^2 |\bar{H}|^2 + (B_\mu \bar{H} H + \text{h.c.}) . \quad (2)$$

An attractive feature of GMSB is that gauginos receive masses at one loop, $m_\lambda \sim \frac{\alpha}{4\pi} \Lambda$, while squarks and sleptons do so only at two loops, $\tilde{m}^2 \sim (\frac{\alpha}{4\pi})^2 \Lambda^2$. Because of the different dimen-

sionalities between fermionic and scalar mass terms, this implies that the gaugino and squark mass scales are of the same order, $m_\lambda \sim \widetilde{m}$. On the other hand, in the case of the Higgs parameters, we find $B_\mu \sim \mu \Lambda$, as parameters with different dimensionalities are generated at the same loop level. Since Λ must be in the range 10–100 TeV to generate appropriate squark and gaugino masses, then either μ is at the weak scale and B_μ violates the naturalness criterion [7], or B_μ is at the weak scale and μ is unacceptably small. We will refer to this puzzle as to the μ -problem in GMSB theories.

We wish to stress that this is only an “aesthetic” problem and not an inconsistency of the theory. It is certainly possible to introduce new interactions able to generate separately both μ and B_μ , but this requires *ad hoc* structures and fine tuning of parameters. On the other hand, our goal here is to propose a solution to the μ -problem in GMSB theories which satisfies the following criteria of naturalness: *i*) the different supersymmetry-breaking Higgs parameters are generated by a single mechanism; *ii*) μ is generated at one loop, while B_μ , m_H^2 , $m_{\widetilde{H}}^2$ are generated at two loops; *iii*) all new coupling constants are of order one; *iv*) there are no new particles at the weak scale.

The paper is organized as follows: in sect. 2 we review the GMSB theories and define the theoretical framework in which we will work. In sect. 3 we present the μ -problem which we will attempt to solve in sect. 4. Finally our results are summarized in sect. 5.

2 The GMSB Theories

In this section we define the set of models which we want to study. We first introduce an *observable sector* which contains the usual quarks, leptons, and two Higgs doublets, together with their supersymmetric partners. Next the theory has a *messenger sector*, formed by some new superfields which transform under the gauge group as a real non-trivial representation. In order to preserve gauge coupling constant unification, we also require that the messengers form complete GUT multiplets. Perturbativity of α_{GUT} at the scale M_{GUT} implies that we can introduce at most $n_5(\mathbf{5} + \overline{\mathbf{5}})$ and $n_{10}(\mathbf{10} + \overline{\mathbf{10}})$ SU_5 representations with $n_5 \leq 4$ for $n_{10} = 0$ and $n_5 \leq 1$ for $n_{10} = 1$ [8]. It should be noticed that, in the minimal SU_5 model, the presence of these new states at scales of about 100 TeV is inconsistent with proton-decay limits and with $b-\tau$ unification [9]. However, these constraints critically depend on the GUT model and will be

dismissed from the point onwards.

Finally the theory contains a *secluded sector*¹. This sector, responsible for the mechanism of supersymmetry breaking in a gauge-invariant direction, has tree-level couplings to the messenger sector, but not to the observable sector. Its effect is to feed two (possibly different) mass scales to the theory: M , the scale of supersymmetric-invariant masses for the messenger superfields, and \sqrt{F} , the effective scale of supersymmetry breaking or, in other words, the mass splittings inside messenger multiplets. We will parametrize the effect of the secluded sector by a mass term in the superpotential

$$W = \bar{\Phi}_i M_{ij} \Phi_j \quad i, j = 1, \dots, n \quad (3)$$

and by a supersymmetry-breaking term in the scalar potential

$$V = \bar{\Phi}_i F_{ij} \Phi_j + \text{h.c.} \quad (4)$$

Here Φ_i and $\bar{\Phi}_i$ are a generic number n of messenger superfields transforming as the representation $\mathbf{r} + \bar{\mathbf{r}}$ under the GUT group. With a standard abuse of notation, we denote the superfields, as in eq. (3), and their scalar components, as in eq. (4), by the same symbol. The interactions in eqs. (3) and (4) can be obtained from tree-level couplings of the messengers Φ and $\bar{\Phi}$ to some superfields X which get Vacuum Expectation Values (VEV) both in their scalar components $\langle X \rangle = M$ and their auxiliary components $\langle F_X \rangle = F$. Supersymmetry breaking can occur dynamically, as in the models of ref. [4], or through perturbative interactions, as in the O’Raifeartaigh model [10] described by the superpotential²

$$W = \lambda X (\bar{\Phi}_1 \Phi_1 - m^2) + M_\Phi (\bar{\Phi}_1 \Phi_2 + \bar{\Phi}_2 \Phi_1) . \quad (5)$$

Indeed, for $\lambda^2 m^2 < M_\Phi^2$, the vacuum of this model is such that $\langle \Phi_i \rangle = \langle \bar{\Phi}_i \rangle = 0$ ($i = 1, 2$) and $\langle F_X \rangle \neq 0$.

Having described the necessary ingredients of the GMSB theories, we can now proceed to compute the feeding of supersymmetry breaking into the observable sector mass spectrum. The mass term for the messenger scalar fields, derived from eqs. (3) and (4) is

$$(\Phi^\dagger \quad \bar{\Phi}) \begin{pmatrix} M^\dagger M & F^\dagger \\ F & M M^\dagger \end{pmatrix} \begin{pmatrix} \Phi \\ \bar{\Phi}^\dagger \end{pmatrix} . \quad (6)$$

¹We introduce this terminology to distinguish this sector from the hidden sector of theories where supersymmetry breaking is mediated by gravity.

²In this specific example the one-loop effective potential fixes $\langle X \rangle = 0$ [11]. However one can easily extend the field content to obtain $\langle X \rangle \neq 0$.

We can now choose a basis in which the matrix M is diagonal and define

$$\varphi = \frac{\Phi + \bar{\Phi}^\dagger}{\sqrt{2}}, \quad \bar{\varphi} = \frac{\bar{\Phi} - \Phi^\dagger}{\sqrt{2}}. \quad (7)$$

In the new basis the scalar messenger mass term becomes

$$(\varphi^\dagger \quad \bar{\varphi}) \begin{pmatrix} M^2 + \frac{F+F^\dagger}{2} & \frac{F^\dagger-F}{2} \\ \frac{F-F^\dagger}{2} & M^2 - \frac{F+F^\dagger}{2} \end{pmatrix} \begin{pmatrix} \varphi \\ \bar{\varphi}^\dagger \end{pmatrix}. \quad (8)$$

We now want to require that there are no one-loop contributions to squark and slepton squared masses proportional to the corresponding hypercharge. These contributions are phenomenologically unacceptable, since they give negative squared masses to some of the squarks and sleptons. They arise from a one-loop contraction of the hypercharge messenger D-term

$$D_\Phi = g' (\Phi^\dagger Y_\Phi \Phi - \bar{\Phi} Y_\Phi \bar{\Phi}^\dagger) = -g' (\bar{\varphi} Y_\Phi \varphi + \varphi^\dagger Y_\Phi \bar{\varphi}^\dagger). \quad (9)$$

If F is Hermitian, then the two sectors φ and $\bar{\varphi}$ do not mix in the mass matrix, and the D-term contributions are not generated up to three loops³. Therefore we require that the secluded sector is such that there exists a basis where M is diagonal and F is Hermitian. For instance, had we chosen different mass parameters for the terms $\bar{\Phi}_1 \Phi_2$ and $\bar{\Phi}_2 \Phi_1$ in eq. (5), there would be no cancellation of one-loop hypercharge D-term contributions to squark and slepton masses, and the model should be discarded.

The next step is the diagonalization of the mass matrices $M^2 \pm F$ for the two sectors φ and $\bar{\varphi}$. After that, the loop computation of the gaugino and squark masses proceeds as discussed in ref. [1]. The result is that, in the limit where the entries of the matrix F are smaller than those of M^2 , the gaugino and scalar masses are⁴

$$m_{\lambda_j} = k_j \frac{\alpha_j}{4\pi} \Lambda_G \left[1 + \mathcal{O}(F^2/M^4) \right], \quad j = 1, 2, 3, \quad (10)$$

$$\widetilde{m}^2 = 2 \sum_{j=1}^3 C_j k_j \left(\frac{\alpha_j}{4\pi} \right)^2 \Lambda_S^2 \left[1 + \mathcal{O}(F^2/M^4) \right], \quad (11)$$

³Indeed, in the absence of the observable sector and for $F = F^\dagger$, the theory is invariant under a parity which transforms $\varphi \rightarrow \varphi$, $\bar{\varphi} \rightarrow -\bar{\varphi}$ and the fermions $\psi \rightarrow \gamma_0 \psi$.

⁴If elements of F were larger than elements of M^2 , the messenger squared mass matrix may develop negative eigenvalues. The requirement that gauge symmetry remains unbroken in the messenger sector justifies the approximation made here.

where $k_1 = 5/3$, $k_2 = k_3 = 1$, and $C_3 = 4/3$ for colour triplets, $C_2 = 3/4$ for weak doublets (and equal to zero otherwise), $C_1 = Y^2$ ($Y = Q - T_3$). The scales Λ_G and Λ_S , in the limit in which $M = M_0 \mathbb{1}$, are given by

$$\Lambda_G = N \frac{\text{Tr} F}{M_0} , \quad (12)$$

$$\Lambda_S = \left(N \frac{\text{Tr} F^2}{M_0^2} \right)^{1/2} , \quad (13)$$

where N is the Casimir of the messenger GUT representation, *e.g.* $N = 1$ for $\mathbf{5} + \overline{\mathbf{5}}$ and $N = 3$ for $\mathbf{10} + \overline{\mathbf{10}}$.

All squarks and sleptons (and analogously all gauginos) receive masses determined by a unique scale Λ_S (or Λ_G). This universality is a consequence of the assumption that the secluded sector contains only GUT singlets and of the fact that the ratio F_{ii}/M_i is not renormalized by gauge interactions.

The values of Λ_G and Λ_S are in general different. If the matrix F is proportional to M , as is the case when the interactions in eqs. (3) and (4) originate from couplings to a single superfield X , then the ratio

$$\Lambda_G/\Lambda_S = \sqrt{Nn} \quad (14)$$

is directly related to the number n of messenger GUT multiplets. However, in general, the ratio Λ_G/Λ_S can be either smaller or larger than one. If $\Lambda_G \neq 0$ then $\Lambda_S \neq 0$, but the converse is not true. For instance, in the model of eq. (5), $\langle X \rangle = 0$ is determined by the one-loop effective potential [11], and $\Lambda_S \neq 0$ while $\Lambda_G = 0$, as a consequence of an exact R-symmetry.

Finally, let us review the present bounds on the different mass scales in the theory. The experimental limits on gluino and right-handed selectron masses require

$$\Lambda_G \gtrsim 16 \text{ TeV} , \quad \Lambda_S \gtrsim 30 \text{ TeV} . \quad (15)$$

This imposes a model-dependent lower bound on the typical scale M .

It is fairly generic that the secluded sector gives rise to an R-axion. If this is the case, one can invoke gravitational interactions to generate a mass for the R-axion. Astrophysical constraints can then be evaded if the typical supersymmetry breaking scale satisfies $\sqrt{F} \lesssim 100$ TeV [12]. Upper bounds on \sqrt{F} can be derived from cosmological considerations. If there is no inflation with low reheating temperature, the constraint on the relic gravitino density requires

$\sqrt{F} < 2 \times 10^3$ TeV [13]. Also if we impose that the lightest supersymmetric particle of the observable sector decays during the first second of the Universe, so that its decay products cannot influence standard nucleosynthesis, then $\sqrt{F} < 10^5$ TeV $(m_{LSP}/100 \text{ GeV})^{5/4}$.

3 The μ -Problem in GMSB Theories

If the μ -term is present in the superpotential in the limit of exact supersymmetry, then naturally it can only be of the order of the Planck scale M_{PL} or some other fundamental large mass scales. We assume therefore that the μ -term is forbidden in the original superpotential but generated, together with B_μ , by the following effective operators

$$\frac{1}{M} \int d^4\theta H \bar{H} X^\dagger, \quad (16)$$

$$\frac{1}{M^2} \int d^4\theta H \bar{H} X X^\dagger. \quad (17)$$

Here X is a superfield which parametrizes the breaking of supersymmetry, as discussed in sect. 2, and M is the messenger mass scale. Replacing X in eqs. (16) and (17) with the VEV of its auxiliary component F , we obtain μ and B_μ terms of the order of Λ and Λ^2 respectively, with $\Lambda = F/M$. In theories where gravity mediates supersymmetry breaking, M has to be identified with M_{PL} and the operators in eqs. (16) and (17) are present in the theory as non-renormalizable interactions. The existence of the operator in eq. (16) requires however a non-minimal Kähler metric [6].

In GMSB theories we want to obtain the operators in eqs. (16) and (17) after integrating out some heavy fields. Since the operators in eqs. (16) and (17) break a Peccei-Quinn symmetry, they cannot be induced by gauge interactions alone. The simplest way to generate a μ -term at the one-loop level is then to couple the Higgs superfields to the messengers in the superpotential:

$$W = \lambda H \Phi_1 \Phi_2 + \bar{\lambda} \bar{H} \bar{\Phi}_1 \bar{\Phi}_2. \quad (18)$$

Assuming that a single superfield X describes the supersymmetry breaking

$$W = X(\lambda_1 \Phi_1 \bar{\Phi}_1 + \lambda_2 \Phi_2 \bar{\Phi}_2), \quad (19)$$

the diagram of fig. 1a gives

$$\mu = \frac{\lambda \bar{\lambda}}{16\pi^2} \Lambda f(\lambda_1/\lambda_2) \left[1 + \mathcal{O}(F^2/M^4) \right], \quad (20)$$

$f(x) = (x \ln x^2)/(1 - x^2)$. However, the couplings in eq. (18) also generate the diagram of fig. 1b, which contributes to B_μ at the one-loop level:

$$B_\mu = \frac{\lambda \bar{\lambda}}{16\pi^2} \Lambda^2 f(\lambda_1/\lambda_2) \left[1 + \mathcal{O}(F^2/M^4) \right]. \quad (21)$$

Finally the diagram of fig. 1c generates the soft-breaking masses m_H^2 and $m_{\bar{H}}^2$. Unexpectedly, the leading order contribution here cancels and the scalar Higgs masses are generated only at higher order $\sim \frac{1}{16\pi^2} F^4/M^6$. However the cancellation is valid only in the simple case of eq. (19) in which the ratios $\Lambda_i = F_{ii}/M_i$ ($i = 1, 2$) for the two messengers are the same. If this is not the case then

$$m_H^2 = \frac{\lambda^2}{16\pi^2} (\Lambda_1 - \Lambda_2)^2 g(\lambda_1/\lambda_2) \left[1 + \mathcal{O}(F^2/M^4) \right], \quad (22)$$

and similarly for $m_{\bar{H}}^2$ with λ replaced by $\bar{\lambda}$; here $g(x) = x^2[(1+x^2) \ln x^2 + 2(1-x^2)]/(1-x^2)^3$.

From eq. (20) and (21) we obtain

$$B_\mu = \mu \Lambda. \quad (23)$$

This problematic relation is the expression of the μ -problem in GMSB theories. It is just a consequence of generating both μ and B_μ at the one-loop level through the same interactions. If the dominant contributions to m_H^2 and $m_{\bar{H}}^2$ come from two-loop gauge and three-loop stop contributions [4],

$$\begin{aligned} m_H^2 &= \frac{3}{2} \left(\frac{\alpha_2}{4\pi} \right)^2 \Lambda^2, \\ m_{\bar{H}}^2 &= m_H^2 \left[1 - \frac{4h_t^2}{3\pi^2} \left(\frac{\alpha_3}{\alpha_2} \right)^2 \ln \left(\frac{\pi}{\alpha_3} \right) \right], \end{aligned} \quad (24)$$

then eq. (23) is actually inconsistent with electroweak symmetry breaking. This is true because the condition for the stability of the Higgs potential, $2|B_\mu| < m_H^2 + m_{\bar{H}}^2 + 2\mu^2$, cannot be satisfied when μ is determined by electroweak symmetry breaking

$$|\mu|^2 = \frac{m_H^2 - m_{\bar{H}}^2 \tan^2 \beta}{\tan^2 \beta - 1} - \frac{m_Z^2}{2}, \quad (25)$$

with $\tan \beta = \langle \bar{H} \rangle / \langle H \rangle$. Extra contributions to m_H^2 and $m_{\bar{H}}^2$ can allow the electroweak symmetry breaking, but only at the price of introducing a considerable fine tuning among parameters.

The alternative to coupling each Higgs superfield separately to the messengers, as done in eq. (18), is to couple the Higgs bilinear $H\bar{H}$ to a heavy singlet superfield S . Of course, for the mechanism to work, S cannot develop a VEV in the supersymmetric limit. Now, possibly

through a coupling to a different singlet N which has tree-level non-zero VEV, a μ -term can be induced at the one-loop level through the graph in fig. 2a [14]. However with an extra insertion of the spurion superfield X , the same interaction gives rise to a B_μ contribution through the graph in fig. 2b and we recover the unwanted relation of eq. (23).

Nevertheless, this is not always the case. Suppose that we couple N to the messengers in the same way as we coupled H to the messengers in eq. (18). From the previous example, we know that the diagram of fig. 1c suffers from a cancellation of the leading contribution in the case of the minimal secluded sector given in eq. (19). This means that also the diagram in fig. 2b vanishes at the leading order. Extra X and X^\dagger or extra N and N^\dagger insertions in the loop of fig. 2b can generate a non-zero B_μ but this will be suppressed by terms $\mathcal{O}(F^2/M^4)$ or $\mathcal{O}(\langle N \rangle^2/M^2)$, if M is larger than the other scales. However we do not consider this mechanism entirely satisfactory, since it relies on an accidental cancellation occurring only for a very specific supersymmetry-breaking structure.

Finally we mention that in the literature the possibility of adding extra light Higgs superfields in GMSB theories has also been considered [4]. If a light singlet S is present with a superpotential

$$W = \lambda H \bar{H} S + \lambda' S^3, \quad (26)$$

then μ and B_μ are generated whenever $\langle S \rangle$ and $\langle F_S \rangle$ are non-zero. However, in GMSB theories, trilinears, bilinears and the soft-mass of S are suppressed with respect to the H and \bar{H} soft masses, and a non-zero VEV for S requires an appreciable fine-tuning [4]. This problem can be overcome but its solution may require additional quark superfields coupled to S in order to induce a large soft mass m_S^2 [4].

4 A Natural Solution to the μ -Problem in GMSB Theories

4.1 The Mechanism

We will describe here a mechanism which satisfies the criteria (i) – (iv) given in sect. 1. We consider the possibility that μ , instead of being generated by the operator (16), arises from the

operator

$$\int d^4\theta H\bar{H}D^2[X^\dagger X]. \quad (27)$$

Here D_α is the supersymmetric covariant derivative. This operator can be generated from the diagram of fig. 3. The crucial point is that a B_μ -term cannot be induced from such diagram even if we added extra X and X^\dagger insertions in the loop of fig. 3. This is because a D^2 acting on any function of X and X^\dagger always produces an antichiral superfield.

Our mechanism requires at least two singlets, S and N , such that only S couples at tree-level to $H\bar{H}$ and to the messengers. We forbid the coupling of N to the messengers and a mass term S^2 in the superpotential to guarantee that the one-loop diagram of fig. 2b does not exist. The diagram of fig. 3 induces the operator

$$\frac{1}{16\pi^2 M^2 M_N^2} \int d^4\theta H\bar{H}D^2[X^\dagger X], \quad (28)$$

where M_N is the mass parameter in the superpotential term $M_N N S$. Notice that the internal line in fig. 3 is an $\langle S S^\dagger \rangle$ propagator which, at small momenta, behaves like \bar{D}^2/M_N^2 [14]. Eq. (28) leads to a μ parameter

$$\mu \sim \frac{1}{16\pi^2} \frac{|F|^2}{M M_N^2} \sim \frac{1}{16\pi^2} \Lambda, \quad (29)$$

for $M_N = \mathcal{O}(\sqrt{F})$.

B_μ can be induced at the two-loop level if the superpotential contains a coupling of the form $N^2 S$. One contribution comes from a diagram analogous to the one shown in fig. 2b, but where the effective coupling between N and X arises at two loops, as shown in fig. 4. Another contribution comes from the diagram of fig. 5, which induces the effective operator

$$\frac{1}{(16\pi^2)^2 M^4 M_N^4} \int d^4\theta H\bar{H} X^\dagger X \bar{D}^2 D^2 [X^\dagger X]. \quad (30)$$

Both contributions generate a B_μ parameter of the correct magnitude

$$B_\mu \sim \left(\frac{1}{16\pi^2} \right)^2 \Lambda^2. \quad (31)$$

In addition to the gauge-induced contributions given in eq. (11), the soft masses m_H^2 and $m_{\bar{H}}^2$ are also generated at two loops by the diagrams in fig. 4.

4.2 A Model

Let us now see how to realize this mechanism in an explicit example. Consider the superpotential

$$W = S(\lambda_1 H \bar{H} + \frac{\lambda_2}{2} N^2 + \lambda \Phi \bar{\Phi} - M_N^2) , \quad (32)$$

together with a secluded sector which, as described in sect. 2, generates a supersymmetric mass M and a supersymmetry-breaking squared mass F for the messengers Φ and $\bar{\Phi}$. For simplicity we assume that the messengers belong to a single $\mathbf{5} + \bar{\mathbf{5}}$ SU_5 representation.

The expression in eq. (32) can be guaranteed by gauge symmetry, a discrete parity of the superfield N , and an R-symmetry. We believe that eq. (32) describes the simplest example in which our mechanism is operative. The tree-level coupling of the singlet S with the messengers in eq. (32) is crucial since it induces, through one-loop radiative corrections, a tadpole for S , which generates $\langle S \rangle \neq 0$ and the desired μ -term. For a tadpole diagram to exist, it is also essential that S does not transform under any discrete or continuous symmetry left unbroken by the secluded sector. In the case of eq. (32), the R-symmetry under which S transform non-trivially is certainly broken (possibly spontaneously) in the secluded sector, or else no gaugino mass is generated.

The tree-level potential is

$$\begin{aligned} V = & \left| \lambda_1 H \bar{H} + \frac{\lambda_2}{2} N^2 + \lambda \Phi \bar{\Phi} - M_N^2 \right|^2 + |S|^2 \left[\lambda_1^2 (|H|^2 + |\bar{H}|^2) + \lambda_2^2 |N|^2 \right] \\ & + |S\lambda + M|^2 (|\Phi|^2 + |\bar{\Phi}|^2) + (F\Phi\bar{\Phi} + \text{h.c.}) . \end{aligned} \quad (33)$$

For $M^2 > F$, a minimum of the potential is at

$$\langle \lambda_1 H \bar{H} + \frac{\lambda_2}{2} N^2 \rangle = M_N^2 , \quad (34)$$

and all other VEVs equal to zero. The two Higgs doublets are massless at tree level and can be viewed as zero modes of the flat direction determined by eq. (34).

One-loop corrections induce a tadpole for the scalar component of S

$$V = \frac{5\lambda}{16\pi^2} \frac{F^2}{M} S + \text{h.c.} , \quad (35)$$

which forces $\langle S \rangle \neq 0$. Inspection of the effective potential shows that there are no runaway directions at large values of S , as discussed in the appendix. Radiative corrections also remove

the degeneracy of the vacuum in eq. (34). An important role is played by the two-loop contributions to the soft masses of N , H , and \bar{H} , obtained from the diagrams of fig. 4 which, for $M > M_N$, give:

$$m_N^2 = 10 \left(\frac{\lambda \lambda_2}{16\pi^2} \right)^2 \frac{F^2}{M^2} , \quad (36)$$

$$\delta m_H^2 = \delta m_{\bar{H}}^2 = 10 \left(\frac{\lambda \lambda_1}{16\pi^2} \right)^2 \frac{F^2}{M^2} . \quad (37)$$

The mass parameters m_H^2 and $m_{\bar{H}}^2$ are then obtained by adding eq. (37) to the gauge and stop contributions given in eq. (24). Including the contributions in eqs. (35)–(37) to the potential in eq. (33) and proceeding in the minimization, we find that the VEV in eq. (34) predominantly lies in the N direction and $\langle S \rangle$ is determined to be

$$\langle S \rangle \simeq -\frac{5}{32\pi^2} \frac{\lambda}{\lambda_2} \frac{F^2}{M_N^2 M} , \quad (38)$$

as long as $\lambda_2(\lambda_1^2 \langle S \rangle^2 + m_H^2) > \lambda_1(\lambda_2^2 \langle S \rangle^2 + m_N^2)$. The two Higgs doublets are the only superfields to remain light. They have the usual low-energy supersymmetry potential with the parameters μ and B_μ given by

$$\mu = \lambda_1 \langle S \rangle = -\frac{5}{32\pi^2} \frac{\lambda \lambda_1}{\lambda_2} \frac{F}{M_N^2} \Lambda , \quad (39)$$

$$B_\mu = \lambda_1 \langle F_S \rangle = -\frac{10\lambda^2 \lambda_1 \lambda_2}{(16\pi^2)^2} \left(1 + \frac{5F^2}{8\lambda_2^2 M_N^4} \right) \Lambda^2 . \quad (40)$$

Equations (39) and (40) confirm in this specific model the estimates, based on general arguments, given in eqs. (29) and (31).

4.3 Pseudo-Goldstone Boson Interpretation

The mechanism that generates $B_\mu \sim \mu^2$, instead of $B_\mu \sim \mu \Lambda$, has a pseudo-Goldstone boson interpretation. Let us modify the previous model by introducing a new gauge singlet \bar{N} and by replacing eq. (32) with

$$W = S(\lambda_1 H \bar{H} + \lambda_2 N \bar{N} + \lambda \Phi \bar{\Phi} - M_N^2) . \quad (41)$$

The results of sect. 4.3 are essentially unaffected. However here, in the limit $\lambda_1 = \lambda_2$, the superpotential has a U(3) symmetry under which $\Sigma \equiv (H, N)$ and $\bar{\Sigma} \equiv (\bar{H}, \bar{N})$ transform as a triplet and an anti-triplet. In the supersymmetric limit, the VEVs of N and \bar{N} break the

$U(3)$ spontaneously to $U(2)$ and the two Higgs doublets are identified with the corresponding Goldstone bosons⁵. Actually they are only pseudo-Goldstone bosons since they get non-zero masses as soon as gauge and Yukawa interactions are switched-on⁶. Nevertheless, at the one-loop level, the relevant part of the effective potential is still $U(3)$ -invariant and one combination of the two Higgs doublets $(H + \bar{H}^\dagger)/\sqrt{2}$, remains exactly massless. Indeed, at one loop, the determinant of the Higgs squared-mass matrix is zero, and

$$B_\mu = |\mu|^2, \quad (42)$$

a general property of models in which the Higgs particles are pseudo-Goldstone bosons [17]. Soft masses for H and \bar{H} are generated at two loops. The contributions from the graphs in fig. 4 preserve the $U(3)$ invariance, but gauge contributions violate the symmetry and the determinant of the Higgs squared-mass matrix no longer vanishes. Nevertheless, we are still guaranteed to obtain a μ and B_μ of the correct magnitude, since eq. (42) is spoiled only by two-loop effects. If we now allow $\lambda_1 \neq \lambda_2$, we will modify eq. (42) but not the property $B_\mu \sim \mu^2$. This provides an explanation, alternative to the one given in sect. 4.1 in terms of effective operators, of the reason why our mechanism can work.

Notice that, although for $\lambda_1 \neq \lambda_2$ the $U(3)$ is no longer a symmetry of the superpotential, the vacuum $\langle \lambda_1 H \bar{H} + \lambda_2 N \bar{N} \rangle = M_N^2$ still has a compact degeneracy isomorphic to $U(3)/U(2)$. This degeneracy of the tree-level vacuum in our model, need not be necessarily an accident of the field structure of the low-energy sector. It may result from an exact $U(3)$, or even $SU(6)$, gauge symmetry spontaneously broken at some high scale in a heavy sector that does not couple with our fields in the superpotential. To be specific, imagine the embedding of our theory in the gauge $SU(3)_C \otimes SU(3)_L \otimes U(1)$ model with Σ and $\bar{\Sigma}$ transforming as triplets and anti-triplets of $SU(3)_L$ respectively. The assignment of quarks and leptons can be easily fixed if we think of $SU(3)_C \otimes SU(3)_L \otimes U(1)$ as a maximal subgroup of $SU(6)$, with quarks and leptons transforming as $\mathbf{15} + \bar{\mathbf{6}} + \bar{\mathbf{6}}$ of $SU(6)$. Suppose that at some high scale there is another sector (*e.g.*, an extra triplet-antitriplet pair $\Sigma', \bar{\Sigma}'$) which does not communicate with $\Sigma, \bar{\Sigma}$ in the superpotential. In such a case the Higgs superpotential has an accidental $U(3)_L \otimes U(3)_L$ approximate symmetry

⁵The idea of interpreting the Higgs doublets as pseudo-Goldstone bosons of some large global symmetry of the superpotential has been introduced in refs. [15, 16].

⁶The theory contains also an exact Goldstone bosons, which corresponds to the spontaneous breaking of the abelian symmetry carried by the N and \bar{N} superfields. However this Goldstone boson has no coupling to ordinary matter.

corresponding to the independent global transformation of $\Sigma, \bar{\Sigma}$ and $\Sigma' \bar{\Sigma}'$ [16]. The crucial point is that a residual global $U(3)_L$ symmetry is left at low energy, if the breaking of the gauge symmetry $SU(3)_L \otimes U(1) \rightarrow SU(2)_L \otimes U(1)_Y$ is induced only by the VEV of $\Sigma' \bar{\Sigma}'$ at the high-energy scale.

5 Conclusions

Let us summarize our results. In sect. 2 we have presented the GMSB theories with a general structure of messenger superfields. We have given the consistency conditions for not generating dangerous negative squark squared masses, and given the general expressions of squark, slepton, and gluino masses. Within our approximations, the ignorance on the messenger sector can be parametrized by the two mass scales Λ_G and Λ_S .

We have shown in sect. 3 that GMSB theories suffer from a μ -problem which has a different aspect than the μ -problem in supergravity. The difficulty here is that once μ is generated by a one-loop diagram, B_μ also arises at the same loop level; this leads to the problematic relation in eq. (23).

In sect. 4 we have considered a mechanism which evades this generic problem. If μ is generated by the effective operator (27), then B_μ is not necessarily induced at the same loop order. We have presented a simple model in which this idea is realized explicitly. This mechanism can naturally lead to an interpretation of the Higgs doublets as pseudo-Goldstone bosons of an approximate global symmetry.

We would like to thank R. Barbieri, S. Dimopoulos, and S. Raby for very useful discussions.

Appendix

In this appendix we want to show that the potential of the model considered in sect. 4.2 does not have runaway directions in the large S region. First note that for sufficiently large values $S > S_c$, such that $|S_c|^2 > M_N^2/\lambda_1, M_N^2/\lambda_2$ and $|S_c\lambda + M|^2 > |\lambda M_N^2 - F|$, the minimum of all fields (other than S) at a given fixed S is at zero. So, for $S > S_c$ the tree-level potential is flat in the S direction and has a constant value $V(S > S_c) = M_N^4$. It is therefore essential to look

at the one-loop corrections to the effective potential

$$V_{eff} = \frac{1}{64\pi^2} (-1)^F \text{Tr } \mathcal{M}^4 \ln \frac{\mathcal{M}^2}{Q^2} . \quad (43)$$

For $S > S_c$, all superfields interacting with S in the superpotential suffer from tree-level mass splittings caused by the non-zero $F_S = -M_N^2$, and therefore contribute to V_{eff} . Evaluating and adding different contributions we get the asymptotic behaviour

$$\begin{aligned} V_{eff}|_{|S| \rightarrow \infty} = & \frac{1}{16\pi^2} \left[2\lambda_1^2 M_N^4 \ln(\lambda_1^2 |S|^2) + \frac{\lambda_2^2}{2} M_N^4 \ln(\lambda_2^2 |S|^2) \right. \\ & \left. + 5(\lambda M_N^2 - F)^2 \ln |\lambda S + M|^2 \right] , \end{aligned} \quad (44)$$

which shows that V_{eff} grows logarithmically at large S .

From the effective potential it is also easy to see that quantum corrections destabilize the tree-level vacuum $\langle S \rangle = 0$. For small S , none of the states $S, \bar{H}, H, \bar{N}, N$ contribute to the effective potential, since there is no tree level mass splitting inside these multiplets. Thus, the only states that contribute are the messengers $\Phi, \bar{\Phi}$. At tree level these states have S -dependent supersymmetric mass-squared $|\lambda S + M|^2$ and mass-splittings $\pm(F - \lambda\lambda_i|S|^2)$ between the real and imaginary parts of their scalar components. The S -dependence of this splitting comes from the F_S -term which, for $S \neq 0$, is equal to $-\lambda_i|S|^2$ where $i = 1$ (or 2) if $|\lambda_2| > |\lambda_1|$ (or $|\lambda_2| < |\lambda_1|$). Evaluating these terms, we get the following result

$$\left(\frac{\partial V_{eff}}{\partial S} \right)_{S=0} = \frac{5}{16\pi^2} \lambda M \sum_{\pm} (M^2 \pm F) \ln \left(1 \pm \frac{F}{M^2} \right) . \quad (45)$$

which, for $M \gg F$ corresponds to the tadpole contribution given in eq. (35).

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Figure Captions

Fig. 1: Superfield Feynman diagrams for generating one-loop contributions to (a) μ , (b) B_μ , and (c) m_H^2 , $m_{\bar{H}}^2$.

Fig. 2: Superfield Feynman diagrams for generating one-loop contributions to (a) μ and (b) B_μ . The encircled cross denotes the VEV of the N scalar component.

Fig. 3: Superfield Feynman diagram for generating a one-loop contribution to μ .

Fig. 4: Superfield Feynman diagrams for generating two-loop contributions to m_N^2 , m_H^2 , and $m_{\bar{H}}^2$.

Fig. 5: Superfield Feynman diagram for generating a two-loop contribution to B_μ .

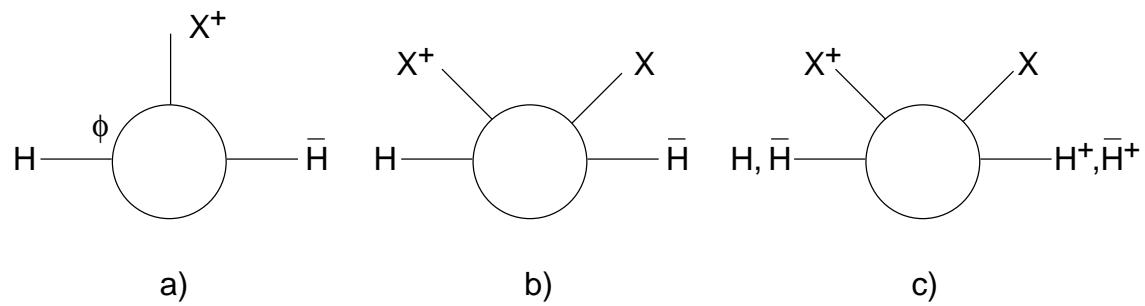


Figure 1

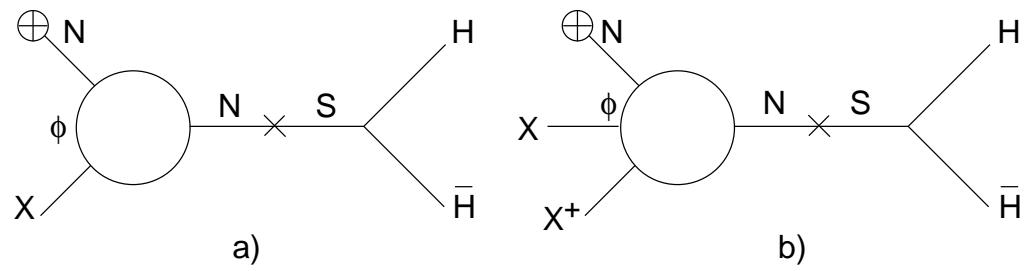


Figure 2

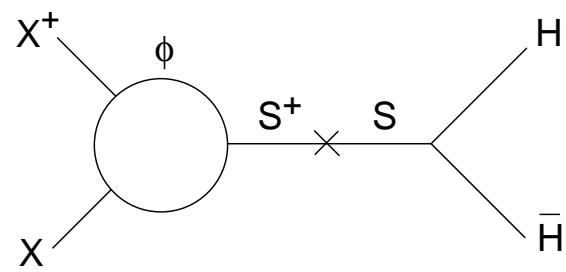


Figure 3

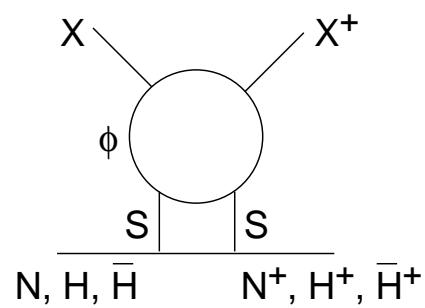


Figure 4

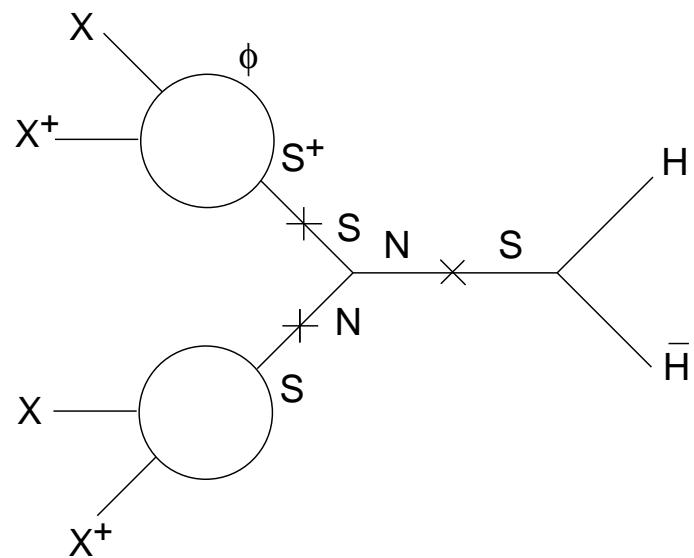


Figure 5